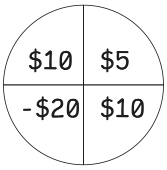
STAT 2290 Exam 3 (Practice), April 2026

- Exam 3 covers Lessons 13-16 on random variables, sampling distributions, and confidence intervals.
- Study lesson examples, homework problems, and problems on this sheet.
- Lesson 17 on confidence interval with unknown $\sigma$ will not appear on Exam 3.

**Some R outputs:** The following R outputs may be useful for various problems.

```
pnorm(-1.5) = 0.0668072012688581
pnorm(-1) = 0.158655253931457
pnorm(-0.5) = 0.308537538725987
```

**Problem 1.** Let $X$ be the monetary amount that you have selected by blindly pointing at one sector of the wheel.
(a) Find the expected value of $X$.
(b) Find the variance of $X$.
(c) Find the standard deviation of $X$.




**Answer:** Do a table of outcomes and their probabilities:

| $X = x$ | **$5** | **$10** | **-$20** |
|---|---|---|---|
| $P(X = x)$ | 1/4 | 2/4 | 1/4 |

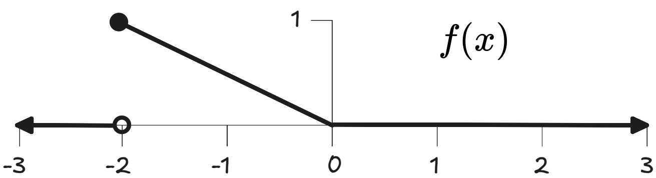
(a) The expected value is $E(X) = (5) \cdot \frac{1}{4} + (10) \cdot \frac{2}{4} + (-20) \cdot \frac{1}{4} = 1.25$

(b) $E(X^2) = (5)^2 \cdot \frac{1}{4} + (10)^2 \cdot \frac{2}{4} + (-20)^2 \cdot \frac{1}{4} = 156.25$

So $\sigma_X^2 = E(X^2) - E(X)^2 = 156.25 - 1.25^2$.

(c) Take the square root of (b) to get your answer.

**Problem 2.** Suppose that a random variable $X$ has PDF $f(x)$ whose graph is shown:




(a) Explain why $f(x)$ is a valid PDF.
(b) Find the CDF of $X$.
(c) Find the median of $X$.
(d) Find the expected value, variance, and standard deviation of $X$.

**Answer:**
(a) $f$ never takes negative value and the total area under $f$ is 1.
(b) Write $f$ as an equation: $f(x) = -x/2$ for $-2 \le x \le 0$ and $f(x) = 0$ otherwise. Then the CDF of $f$ is

- $F(x) = 0$ if $x < -2$,
- $F(x) = 1$ if $x > 0$,
- $F(x) = \int_{-2}^{x} -x/2 \, dx = \left[\frac{-x^2}{4}\right]_{-2}^{x} = 1 - \frac{x^2}{4}$ if $-2 \leq x \leq 0$

(c) $0.5 = F(x) = 1 - \frac{x^2}{4}$ gives $x = \pm\sqrt{2}$ but only $-\sqrt{2}$ split our PDF into two equal parts.
(d) (Corrected on 2026-04-15):
We only do expected value of $X$:

$$E(X) = \int_{-\infty}^{\infty} x f(x) \, dx = \int_{-2}^{0} x(\frac{-x}{2}) \, dx = \int_{-2}^{0} -\frac{x^2}{2} \, dx = \left[-\frac{x^3}{6}\right]_{-2}^{0} = -\frac{4}{3}$$

**Problem 3.** Suppose that a population of common fish have weights which are normally distributed with $\mu = 10$ lbs. and $\sigma = 3$ lbs.
Find the probability that a sample of size $n = 9$ taken from this population produces a sample mean of $\leq 9$ lbs.

**Answer:**
To do calculations, we check the conditions from the "facts about sampling distribution for sample mean" first.

- 10% Condition: The population of common fish surely has size at least $10 \cdot n = 10 \cdot 9 = 90$ so this is satisfied.
- The population is normally distributed ... OK.

We can now use any fact about sampling distribution of $\bar{x}$.

- $\mu_{\bar{x}} = \mu = 10$
- $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{3}{\sqrt{9}} = 1$

So $P(\bar{x} \leq 9) =$ `pnorm(9, mean=10, sd=1) = pnorm((9-10)/1)=pnorm(-1)` which from the top of the page we know is 0.1586 or 15.9%